RESEARCH STATEMENT

KEITH PENROD

UNDERGRADUATE RESEARCH

As an undergraduate student, I researched a few minimization problems called Steiner point problems. The question is given a (finite) set of fixed points, what is the least-length path network to connect all of these points? This must be studied in a geodesic space, for the apparent reason. My first published paper was on the 4-point Steiner problem in the hyperbolic plane [12]. I also did work on the 3-point problem in the flat torus.

1. Master Research

During my time as an MS student, I studied the Hawaiian Earring Group, which has been studied heavily in the past (see [4], [6], [8], [9], [16]). In particular, I expanded on the concept of infinite multiplication that can be seen exhibited in this extraordinary group. The Hawaiian Earring Group can be thought of, combinatorially, as the set of all words of countable length on the alphabet \{a_1, a_2, \ldots\} (together with their formal inverses) where each letter is used only finitely often. Cancellation is done not just one pair at a time, but an infinite number of letters can cancel as long as they are paired with an inverse without any letters obstructing the pairing.

This concept of infinite multiplication was used to define infinite multiplication structures on other groups, namely braid groups (see [10]), mapping class groups (see [11]), and symmetric groups (see [2]). These groups have been studied, as seen in the references given and many others, but my research was to study them in the light of an infinitely multiplicative structure. In specific, to answer the question of when it is possible to multiply infinitely many group elements together.

The theory is built around endowing the group with a topology and then defining the infinite multiplication using limits of partial sub-products. For example, given a group \( G \) with a topology and a sequence \( g_1, g_2, \ldots \in G \), one could ask whether the sequence \( \{g_1, g_1g_2, g_1g_2g_3, \ldots\} \) converges in \( G \). If so, define this to be the product \( g_1g_2g_3\cdots\).

In my master’s thesis, I examined only a few specific groups and how an infinite product structure may help to understand these groups better. During more recent years, I have tried to define in general what an infinite product structure on a group might be and develop general theory for studying the field. However, not much progress has been made and not much motivation for doing so has been found, so the project has been set aside, perhaps temporarily.

An electronic version of my master’s thesis is available at this address.
http://hdl.lib.byu.edu/1877/etd1977

DOCTORAL RESEARCH

As a PhD student, my research has focused on the concept of generalizing homotopy and homology groups. In classical homotopy theory, a homotopy group is calculated by mapping spheres, created by the one-point compactification of Euclidean space, into a topological
space and then taking homotopy classes. While this is a powerful tool, and much research is being done in the area, another approach is to map in objects other than spheres.

One such generalization, which I have been working on, is that of mapping “big spheres” into a topological space. Instead of a product of real intervals \([0, 1]\), one can use a product of “big intervals”. A “big interval” is a connected, compact, totally-ordered set. This has all of the nice properties of the real interval, aside from the fact that it is separable. In fact, it may not even be first countable.

In [4], Cannon and Conner defined the “big fundamental group” of a topological space. My doctoral dissertation has expanded on the theory they developed. In their original definition of the big fundamental group, they take a “big path” to be a function from any big interval, which is unwieldy, since big intervals exist for every cardinality, hence sometimes the collection of homotopy classes of big loops isn’t even a set, it’s a proper class.

In my dissertation, I define specific big fundamental groups for any given cardinality. I construct a homogeneous big interval for arbitrarily large cardinalities, much like the unit interval. I extend the idea to higher dimensions, creating big homotopy groups.

I am excited to continue research in these areas and find exciting and useful ways to apply the results to interesting questions in the fields related to topology and algebra.

An electronic form of my dissertation is available at this address. http://trace.tennessee.edu/cgi/viewcontent.cgi?article=2847&context=utk_graddiss

Future Research. I plan to continue my research in the field of algebraic topology. I am interested in wild topological spaces and the algebraic methods that have been developed to distinguish them. I am interested in learning more about the fundamental groups of wild spaces, about homology and cohomology, and about generalizations of these algebraic groups and rings in the pursuit of studying more wild spaces.

REFERENCES

3. George M Bergman, Two statements about infinite products that are not quite true, Groups, Rings, and Algebras 420 (2006), 35–58.
16. Erin Summers, *There is no surjective map from the hawaiian earring group to the double hawaiian earring group*, Master’s thesis, Brigham Young University, 2002. 1